Profiles beams optimization problems for remote radiation therapy as multicriterion problem

D.A.Rubinsky, V.A.Klimanov, S.G.Klimanov, A.V.Krianev

Moscow State Engineering Physical Institute (Technical University), Moscow, Russia

Email: den@rosnet.ru

Introduction.

Profiles beams optimization problem for tumors in remote radiation therapy has been researched in lot of papers during last years. This paper deals with formulation of profiles intensity optimization problem as a multi-criterion one and application of finite-size pencil beam algorithm and large-scaled elements method for its numeric solution.

Optimization problem formulation.

Basically, profiles radiation treatment optimization problem belongs to the class of multi-criterion problems as soon as during optimization procedure several risk organs are to be taken into account and radiation treatment quality for each one is determined, for example, by doze mean deviation from zero doze

\[ I_k = \frac{1}{n_k} \sum_{i \in V_k} d_i^2, \quad k=1,\ldots,m \]  

\[ (1) \]

At the same time radiation treatment quality for tumor is determined by doze mean deviation from the fixed doze, defined by the patient’s clinical tests

\[ I_0 = \frac{1}{n_0} \sum_{i \in V_0} (d_i - d_0)^2 \]  

\[ (2) \]

where

- \( d_i \) - doze in the i-th voxel;
- \( V_k \) - volume of the k-th risk organ, \( k=1,\ldots,m \);
- \( n_k \) - number of control voxels of k-th risk organ;
- \( V_0 \) - tumor volume;
- \( n_0 \) - number of tumor voxels;
- \( m \) - number of risk organs.

Minimization of objective functions \( I_k, k=0,1,\ldots,m \) combination is the essence of optimization problem. Thus and so we say this problem to be multi-criterion one.

Optimal solution for multi-criterion problem is known to be found out among Pareto solutions in which optimal compromise between concerned criteria is set.

Minimization of linear combinations of initial criteria is one of the ways to find Pareto solutions. That’s why we use physical objective functions

\[ I = (1-\alpha) \sum_{k=1}^{m} \frac{W_k}{n_k} \sum_{i \in V_k} d_i^2 + \frac{\alpha}{n_0} \sum_{i \in V_0} (d_i - d_0)^2 \]  

\[ (3) \]

where

- \( 0 < \alpha \leq 1 \), \( W_k > 0 \), \( \sum_{k=1}^{m} W_k = 1 \), \( \alpha \) - radiation compromise coefficient between tumor and risk organs;
- \( W_k \) - radiation compromise coefficients between risk organs.

The appropriate compromise coefficients meanings are adjusted with account to the maximum admissible meanings for patient’s risk organs.

To formulate and solve the initial profile beam fluence optimization problem numerically connection between vector \( \psi = (\psi_1,\ldots,\psi_N)^T \) of fluence control...
parameters $\psi_j$ and doze vector $d=(d_1,\ldots,d_n)^T$ of control voxels is required

$$D\psi = d,$$  \hspace{1cm} (4)

where $D=(D_{ij}) - (nxN)$ doze matrix of influence; $D_{ij}$ – doze in the i-th voxel irradiated by the j-th field segment (the j-th group of pixels); $n$- total number of control voxels ($n=n_0+n_1+\ldots+n_m$); $N$- number of formed large-scaled elements (groups) determining energy profile beam fluence.

**Optimization problem solution method.**

Large-scaled elements method gives us an opportunity to decrease number of control parameters essentially by combining small-sized pixels in appropriate large-scaled elements (groups) with uniform radiation intensity control parameter per group. Reduction of control parameters number should not cause the essential loss of accuracy for optimization problem solution.

Various schemes may be applied to form large-scaled elements, for example, conformal mapping of the tumor volume $V_0$ projection border on the irradiation plane. Rings of conformal mapping of tumor projection border will be large-scaled elements. To form large-scaled elements other schemes also may be used including multi-step processes.

To solve optimization problem vector-matrix representation of objective function is convenient.

$$I(d) = (1-\alpha)\sum_{k=1}^{m} \frac{W_k}{n_k}(P_k d,d) + \frac{\alpha}{n_0} (P_0 d,d) - \frac{2\alpha}{n_0} d_0 (p_0 d),$$  \hspace{1cm} (5)

where $p_0=(0,1,\ldots,1,0)^T$- n-dimension vector with the i-th component set to 1 whether the i-th voxel belongs to tumor volume $V_0$ and to 0 otherwise; $P_0=\text{diag}(p_0)$- scalar matrix. Similarly vectors $p_k$ and scalar matrices $P_k=\text{diag}(p_k)$ corresponding to risk organs are defined.

Substituting doze vector $d$ in (5) for radiation fluence vector $\psi$ in (4) we have the following representation of objective function

$$I(\psi) = \frac{1}{2} (C\psi,\psi) - (b,\psi),$$  \hspace{1cm} (6)

where $C=(C_{ij})$- symmetric positively definite matrix and $b=(b_1,\ldots,b_n)^T$- vector.

To minimize objective function $I(\psi)$ with limitations $0 \leq \psi_j \leq \psi_{\text{max}}$ we use the following iterative process

$$\phi_j^{(r+1)} = \frac{1}{C_{jj}} (b_j - \sum_{i=1}^{j-1} C_{ij}\phi_i^{(r+1)} - \sum_{i=j+1}^{N} C_{ij}\phi_i^{(r)}),$$  \hspace{1cm} (7)

$$\phi_j^{(r+1)} = \max\{0, \phi_j^{(r+1)}\}, \psi_j^{(r+1)} = \min\{\phi_j^{(r+1)}, \psi_{\text{max}}\}, j=1,\ldots,N$$

Doze matrix $D$ calculations and doze distributions in voxels are provided with accordance to the pencil beam or the finite-size pencil beam models. These models use energy deposition kernel calculations results for pencil monoenergy sources in bandwidth from 0.1 MeV up to 30.0 MeV.

**Conclusions.**

Obtained numerical results allow to conclude the following:

1. Iterative method (7) is stable and has a good convergence to optimization problem solution.
2. Calculation method for doze distributions provides sufficient accuracy level and acceptable machine time.
3. Large-scaled elements method allows essentially decrease number of control parameters for optimization problem without essential loss of accuracy for optimization problem result.
4. Suggested scheme for doze treatment optimization provides the maximum possible uniform doze distribution and required doze levels in tumor and risk organs.

This work was carried out under financial support of ISTC in the frame of project #1079.
References.

DVH representation
Proton beam radiation therapy is used routinely for certain types of cancer, but still need more study in treating others. It requires highly specialized equipment and is not widely available. Some of the techniques used in proton treatment can also expose the patient to neutrons (see below). Neutron beams are used for some cancers of the head, neck, and prostate and for certain inoperable tumors. The radiation therapist or radiation therapy technologist operates the radiation equipment and positions you for treatment. A radiation therapy nurse is a registered nurse with special training in cancer treatment. He or she will be able to give you information about your radiation treatment and advice on how to deal with any side effects you might have. Beam angle optimization (BAO) is a computationally intensive problem for a number of reasons. First, the search space of the solutions is huge, requiring enumeration of all possible beam orientation combinations. For example, when choosing 4 angles out of 36 candidate beam angles. In the FMO formulation, each beam angle can be treated as hundreds of smaller beamlets, each of which having its own radiation intensity (called a fluence). The modulation of these fluences for the beamlets in a set of beams allows for precise control of radiation delivery to the patient. To accelerate each iteration of dose calculation, a strategy similar to that reported by Djajaputra et al.